$\boxed{8.1}$

#23 $\int \frac{x^2}{x-1}\,dx$

$= \int \left[x+1 + \frac{1}{x-1}\right] dx$

$= \int x\,dx + \int 1\,dx + \int \frac{1}{x-1}\,dx$

$= \frac{x^2}{2} + x + \int \frac{1}{u}\,du$

$= \frac{x^2}{2} + x + \ln|u| + c$

$\boxed{= \frac{x^2}{2} + x + \ln|x-1| + c}$

long division

$$\begin{array}{r} x + 1 \quad R\,1 \\ x-1 \enclose{longdiv}{x^2 + 0 \cdot x + 0} \\ -(x^2 - x) \\ \hline x + 0 \\ -(x - 1) \\ \hline 1 \end{array}$$

$\boxed{\frac{x^2}{x-1} = x+1 + \frac{1}{x-1}}$

let $u = x-1$

$\frac{du}{dx} = 1$

$du = x$

$\boxed{8.1}$

#37 $\displaystyle\int \frac{\ln(x^2)}{x}\,dx$

SDWK

let $u = \ln(x)$

$\dfrac{du}{dx} = \dfrac{1}{x}$

$x\,du = dx$

$= 2\displaystyle\int \frac{\ln(x)}{x}\,dx$

$= 2\displaystyle\int \frac{u}{x}(x\,du)$

$= 2\displaystyle\int u\,du$

$= 2\left[\dfrac{u^2}{2}\right] + C$

$= u^2 + C$

$\boxed{= [\ln(x)]^2 + C}$

$\boxed{8.2}$

#73 $\int 1\cdot\cos(\ln(x))\,dx$

"Parts"

$\int u\,dv = uv - \int v\,du$

$= [\cos(\ln(x))][x] - \int (x)\left(-\frac{\sin(\ln(x))}{x}\,dx\right)$

Let
$u = \cos(\ln(x))$
$\frac{du}{dx} = -\sin(\ln(x))\cdot\frac{1}{x}$
$du = -\frac{\sin(\ln(x))}{x}\,dx$

$\frac{dv}{dx} = 1$
$dv = dx$
$v = x$

$= x\cos(\ln(x)) + \int \sin(\ln(x))\,dx$

$= x\cos(\ln(x)) + \int 1\cdot\sin(\ln(x))\,dx$

Loop!

$= x\cos(\ln(x)) + [\sin(\ln(x))][x] - \int [x]\left(\frac{\cos(\ln(x))}{x}\,dx\right)$

$= x\cos(\ln(x)) + x\sin(\ln(x)) - \int \cos(\ln(x))\,dx$

let $u = \sin(\ln(x))$
$\frac{du}{dx} = \cos(\ln(x))\cdot\frac{1}{x}$
$du = \frac{\cos(\ln(x))}{x}\,dx$

$\frac{dv}{dx} = 1$
$\int dv = \int dx$
$v = x$

$\int \cos(\ln(x))\,dx = x\cos(\ln(x)) + x\sin(\ln(x)) - \int \cos(\ln(x))\,dx$
$+ \int \cos(\ln(x))\,dx = \qquad + \int \cos(\ln(x))\,dx$

$2\int \cos(\ln(x))\,dx = x\cos(\ln(x)) + x\sin(\ln(x)) + C$

$\frac{1}{2}\cdot 2\int \cos(\ln(x))\,dx = \frac{1}{2}\left[x\cos(\ln(x)) + x\sin(\ln(x))\right] + C$

$$\boxed{\int \cos(\ln(x))\,dx = \frac{1}{2}\left[x\cos(\ln(x)) + x\sin(\ln(x))\right] + C}$$

"Parts"

$$\boxed{\int u\,dv = uv - \int v\,du}$$

[8.3]

#29 $\int \sec^3(\pi x)\,dx$

$= \int \sec(\pi x)\cdot\sec^2(\pi x)\,dx$

$= [\sec(\pi x)][\frac{1}{\pi}\tan(\pi x)] - \int [\frac{1}{\pi}\tan(\pi x)][\pi\sec(\pi x)\tan(\pi x)\,dx]$

$= \frac{1}{\pi}\sec(\pi x)\tan(\pi x) - \int \tan^2(\pi x)\sec(\pi x)\,dx$

$= \frac{1}{\pi}\sec(\pi x)\tan(\pi x) - \int [\sec^2(\pi x) - 1]\sec(\pi x)\,dx$

$= \frac{1}{\pi}\sec(\pi x)\tan(\pi x) - \int \sec^3(\pi x)\,dx + \int \sec(\pi x)\,dx$ ***

Loop!

$= \frac{1}{\pi}\sec(\pi x)\tan(\pi x) - \int \sec^3(\pi x)\,dx + \frac{1}{\pi}\ln|\sec(\pi x) + \tan(\pi x)|$ ***

let $u = \sec(\pi x)$

$\frac{du}{dx} = \sec(\pi x)\tan(\pi x)\cdot\frac{d}{dx}(\pi x)$

$\frac{du}{dx} = \sec(\pi x)\tan(\pi x)\cdot\pi$

$du = \pi\sec(\pi x)\tan(\pi x)\,dx$

$\frac{dv}{dx} = \sec^2(\pi x)$

$\int dv = \int \sec^2(\pi x)\,dx$

$v = \frac{1}{\pi}\tan(\pi x)$

$\tan^2\theta = \sec^2\theta - 1$

$\int \sec^3(\pi x)\,dx = \frac{1}{\pi}\sec(\pi x)\tan(\pi x) + \frac{1}{\pi}\ln|\sec(\pi x) + \tan(\pi x)| - \int \sec^3(\pi x)\,dx$

$+ \int \sec^3(\pi x)\,dx \quad = \quad + \int \sec^3(\pi x)\,dx$

$\frac{1}{2}\cdot 2\int \sec^3(\pi x)\,dx = \frac{1}{2}\cdot\frac{1}{\pi}[\sec(\pi x)\tan(\pi x) + \ln|\sec(\pi x) + \tan(\pi x)|] + C$

$$\boxed{\int \sec^3(\pi x)\,dx = \frac{1}{2\pi}[\sec(\pi x)\tan(\pi x) + \ln|\sec(\pi x) + \tan(\pi x)|] + C}$$

*** $\int \sec(\pi x)\,dx = \int \sec(z)\cdot\frac{dz}{\pi}$

$= \frac{1}{\pi}\cdot\ln|\sec(z) + \tan(z)|$

$= \frac{1}{\pi}\cdot\ln|\sec(\pi x) + \tan(\pi x)|$

$z = \pi x$

$\frac{dz}{dx} = \pi$

$\frac{dz}{\pi} = dx$

$\boxed{8.4}$

#39 $$\int \frac{1}{4+4x^2+x^4}\,dx$$

$$= \int \left(\frac{\cos^4(\theta)}{4}\right)\left(\sqrt{2}\sec^2(\theta)\,d\theta\right)$$

$$= \frac{\sqrt{2}}{4}\int \frac{\cos^4(\theta)}{1}\cdot\frac{1}{\cos^2\theta}\,d\theta$$

$$= \frac{\sqrt{2}}{4}\int \cos^2(\theta)\,d\theta$$

$$= \frac{\sqrt{2}}{4}\int \left[\frac{1+\cos(2\theta)}{2}\right]d\theta$$

$$= \frac{\sqrt{2}}{8}\int [1+\cos(2\theta)]\,d\theta$$

$$= \frac{\sqrt{2}}{8}\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$$

$$= \frac{\sqrt{2}}{8}\theta + \frac{\sqrt{2}}{16}\sin(2\theta) + C$$

$$= \frac{\sqrt{2}}{8}\left[\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right] + \frac{\sqrt{2}}{16}\left[2\sin(\theta)\cos(\theta)\right] + C$$

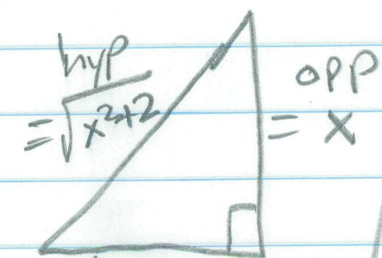
$$= \frac{\sqrt{2}}{8}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{\sqrt{2}}{8}\left[\frac{x}{\sqrt{x^2+2}}\right]\left[\frac{\sqrt{2}}{\sqrt{x^2+2}}\right] + C$$

$$\boxed{= \frac{\sqrt{2}}{8}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{x}{4(x^2+2)} + C}$$

** $\theta = \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

SDWK

$$x^4+4x^2+4 = (x^2+2)^2$$



adj $= \sqrt{2}$

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{x}{\sqrt{x^2+2}}$

$\tan\theta = \frac{opp}{adj}$

$$\tan\theta = \frac{x}{\sqrt{2}}$$

$$\sqrt{2}\tan\theta = x \quad **$$

$$\sqrt{2}\cdot\frac{d}{d\theta}[\tan\theta] = \frac{d}{d\theta}[x]$$

$$\sqrt{2}\cdot\sec^2\theta = \frac{dx}{d\theta}$$

$$\sqrt{2}\cdot\sec^2\theta\,d\theta = dx$$

$\frac{adj}{hyp} = \frac{\sqrt{2}}{\sqrt{x^2+2}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{x^2+2}}$

$$\cos^2\theta = \left(\frac{\sqrt{2}}{\sqrt{x^2+2}}\right)^2 = \frac{2}{x^2+2}$$

$$(\cos^2\theta)^2 = \left(\frac{2}{x^2+2}\right)^2 = \frac{4}{(x^2+2)^2}$$

$$\cos^4\theta = \frac{4}{(x^2+2)^2}$$

$$\frac{\cos^4\theta}{4} = \frac{1}{(x^2+2)^2}$$